"Save John Connor": solutions

1. $P(\neg e) = .9$
2. $P(\neg r \mid e) = .2$
3. $P(\neg r \wedge e) = .2 \cdot .1 = .02$
4. $P(asylum \mid \neg r \wedge e) = .5$
5. $P(asylum \wedge \neg r \wedge e) = .5 \cdot .2 \cdot .1 = .01$
6. $P(\neg k \wedge \neg n \wedge atlarge \wedge r \wedge e) = .1 \cdot .8 \cdot .2 \cdot .8 \cdot .1 = .00128$
7.

$$\begin{aligned}
P(asylum) = {} & P(asylum \mid r \wedge e) \cdot P(r \wedge e) + \\
& P(asylum \mid r \wedge \neg e) \cdot P(r \wedge \neg e) + \\
& P(asylum \mid \neg r \wedge e) \cdot P(\neg r \wedge e) + \\
& P(asylum \mid \neg r \wedge \neg e) \cdot P(\neg r \wedge \neg e) \\
= {} & .6 \cdot .8 \cdot .1 + \\
& .2 \cdot .001 \cdot .9 + \\
& .5 \cdot .2 \cdot .1 + \\
& .3 \cdot .999 \cdot .9 \\
= {} & .32791
\end{aligned}$$

And, as long as we're at it...

$$\begin{aligned}
P(foster) = {} & P(foster \mid r \wedge e) \cdot P(r \wedge e) + \\
& P(foster \mid r \wedge \neg e) \cdot P(r \wedge \neg e) + \\
& P(foster \mid \neg r \wedge e) \cdot P(\neg r \wedge e) + \\
& P(foster \mid \neg r \wedge \neg e) \cdot P(\neg r \wedge \neg e) \\
= {} & P(foster \mid r \wedge e) \cdot P(r \mid e) \cdot P(e) + \\
& P(foster \mid r \wedge \neg e) \cdot P(r \mid \neg e) \cdot P(\neg e) + \\
& P(foster \mid \neg r \wedge e) \cdot P(\neg r \mid e) \cdot P(e) + \\
& P(foster \mid \neg r \wedge \neg e) \cdot P(\neg r \mid \neg e) \cdot P(\neg e) \\
= {} & .2 \cdot .8 \cdot .1 + \\
& .2 \cdot .001 \cdot .9 + \\
& .2 \cdot .2 \cdot .1 + \\
& .3 \cdot .999 \cdot .9 \\
= {} & .28991
\end{aligned}$$

$$
\begin{aligned}
P(atlarge) &= P(atlarge \mid r \wedge e) \cdot P(r \wedge e) + \\
&\quad P(atlarge \mid r \wedge \neg e) \cdot P(r \wedge \neg e) + \\
&\quad P(atlarge \mid \neg r \wedge e) \cdot P(\neg r \wedge e) + \\
&\quad P(atlarge \mid \neg r \wedge \neg e) \cdot P(\neg r \wedge \neg e) \\
&= P(atlarge \mid r \wedge e) \cdot P(r \mid e) \cdot P(e) + \\
&\quad P(atlarge \mid r \wedge \neg e) \cdot P(r \mid \neg e) \cdot P(\neg e) + \\
&\quad P(atlarge \mid \neg r \wedge e) \cdot P(\neg r \mid e) \cdot P(e) + \\
&\quad P(atlarge \mid \neg r \wedge \neg e) \cdot P(\neg r \mid \neg e) \cdot P(\neg e) \\
&= .2 \cdot .8 \cdot .1 + \\
&\quad .6 \cdot .001 \cdot .9 + \\
&\quad .3 \cdot .2 \cdot .1 + \\
&\quad .4 \cdot .999 \cdot .9 \\
&= .38218
\end{aligned}
$$

So before we get any evidence, our prior estimates are about a 33% chance of the T-1000 being at the insane asylum, a 29% chance of its being at John's foster parents', and a 38% of its being anywhere else.

8. $P(asylum \mid n \wedge \neg k)$ can be computed in a couple of different ways: computing and dividing by the marginal, or using an α normalization constant. I'll do both. In either case, the numerator is $P(asylum \wedge n \wedge \neg k)$, computed as follows:

$$
\begin{aligned}
P(asylum \wedge n \wedge \neg k) &= P(asylum \wedge n \wedge \neg k \wedge e \wedge r) + \\
&\quad P(asylum \wedge n \wedge \neg k \wedge e \wedge \neg r) + \\
&\quad P(asylum \wedge n \wedge \neg k \wedge \neg e \wedge r) + \\
&\quad P(asylum \wedge n \wedge \neg k \wedge \neg e \wedge \neg r) \\
&= P(e) \cdot P(r \mid e) \cdot P(asylum \mid r \wedge e) \cdot P(n \mid asylum) \cdot P(\neg k \mid asylum \wedge n) + \\
&\quad P(e) \cdot P(\neg r \mid e) \cdot P(asylum \mid \neg r \wedge e) \cdot P(n \mid asylum) \cdot P(\neg k \mid asylum \wedge n) + \\
&\quad P(\neg e) \cdot P(r \mid \neg e) \cdot P(asylum \mid r \wedge \neg e) \cdot P(n \mid asylum) \cdot P(\neg k \mid asylum \wedge n) + \\
&\quad P(\neg e) \cdot P(\neg r \mid \neg e) \cdot P(asylum \mid \neg r \wedge \neg e) \cdot P(n \mid asylum) \cdot P(\neg k \mid asylum \wedge n) \\
&= .1 \cdot .8 \cdot .6 \cdot .2 \cdot .05 + \\
&\quad .1 \cdot .2 \cdot .5 \cdot .2 \cdot .05 + \\
&\quad .9 \cdot .001 \cdot .2 \cdot .2 \cdot .05 + \\
&\quad .9 \cdot .999 \cdot .3 \cdot .2 \cdot .05 \\
&= .0032791
\end{aligned}
$$

(a) Computing and dividing by the marginal. First, compute the marginal:

$$
\begin{aligned}
P(n \land \neg k) = {} & P(asylum \land n \land \neg k \land e \land r) + \\
& P(asylum \land n \land \neg k \land e \land \neg r) + \\
& P(asylum \land n \land \neg k \land \neg e \land r) + \\
& P(asylum \land n \land \neg k \land \neg e \land \neg r) + \\
& P(foster \land n \land \neg k \land e \land r) + \\
& P(foster \land n \land \neg k \land e \land \neg r) + \\
& P(foster \land n \land \neg k \land \neg e \land r) + \\
& P(foster \land n \land \neg k \land \neg e \land \neg r) + \\
& P(atlarge \land n \land \neg k \land e \land r) + \\
& P(atlarge \land n \land \neg k \land e \land \neg r) + \\
& P(atlarge \land n \land \neg k \land \neg e \land r) + \\
& P(atlarge \land n \land \neg k \land \neg e \land \neg r) \\
= {} & P(e) \cdot P(r \mid e) \cdot P(asylum \mid r \land e) \cdot P(n \mid asylum) \cdot P(\neg k \mid asylum \land n) + \\
& P(e) \cdot P(\neg r \mid e) \cdot P(asylum \mid \neg r \land e) \cdot P(n \mid asylum) \cdot P(\neg k \mid asylum \land n) + \\
& P(\neg e) \cdot P(r \mid \neg e) \cdot P(asylum \mid r \land \neg e) \cdot P(n \mid asylum) \cdot P(\neg k \mid asylum \land n) + \\
& P(\neg e) \cdot P(\neg r \mid \neg e) \cdot P(asylum \mid \neg r \land \neg e) \cdot P(n \mid asylum) \cdot P(\neg k \mid asylum \land n) + \\
& P(e) \cdot P(r \mid e) \cdot P(foster \mid r \land e) \cdot P(n \mid foster) \cdot P(\neg k \mid foster \land n) + \\
& P(e) \cdot P(\neg r \mid e) \cdot P(foster \mid \neg r \land e) \cdot P(n \mid foster) \cdot P(\neg k \mid foster \land n) + \\
& P(\neg e) \cdot P(r \mid \neg e) \cdot P(foster \mid r \land \neg e) \cdot P(n \mid foster) \cdot P(\neg k \mid foster \land n) + \\
& P(\neg e) \cdot P(\neg r \mid \neg e) \cdot P(foster \mid \neg r \land \neg e) \cdot P(n \mid foster) \cdot P(\neg k \mid foster \land n) + \\
& P(e) \cdot P(r \mid e) \cdot P(atlarge \mid r \land e) \cdot P(n \mid atlarge) \cdot P(\neg k \mid atlarge \land n) + \\
& P(e) \cdot P(\neg r \mid e) \cdot P(atlarge \mid \neg r \land e) \cdot P(n \mid atlarge) \cdot P(\neg k \mid atlarge \land n) + \\
& P(\neg e) \cdot P(r \mid \neg e) \cdot P(atlarge \mid r \land \neg e) \cdot P(n \mid atlarge) \cdot P(\neg k \mid atlarge \land n) + \\
& P(\neg e) \cdot P(\neg r \mid \neg e) \cdot P(atlarge \mid \neg r \land \neg e) \cdot P(n \mid atlarge) \cdot P(\neg k \mid atlarge \land n) \\
= {} & .1 \cdot .8 \cdot .6 \cdot .2 \cdot .05 + \\
& .1 \cdot .2 \cdot .5 \cdot .2 \cdot .05 + \\
& .9 \cdot .001 \cdot .2 \cdot .2 \cdot .05 + \\
& .9 \cdot .999 \cdot .3 \cdot .2 \cdot .05 + \\
& .1 \cdot .8 \cdot .2 \cdot .9 \cdot .99 + \\
& .1 \cdot .2 \cdot .2 \cdot .9 \cdot .99 + \\
& .9 \cdot .001 \cdot .2 \cdot .9 \cdot .99 + \\
& .9 \cdot .999 \cdot .3 \cdot .9 \cdot .99 + \\
& .1 \cdot .8 \cdot .2 \cdot .2 \cdot .05 + \\
& .1 \cdot .2 \cdot .3 \cdot .2 \cdot .05 + \\
& .9 \cdot .001 \cdot .6 \cdot .2 \cdot .05 + \\
& .9 \cdot .999 \cdot .4 \cdot .2 \cdot .05 \\
= {} & .26541
\end{aligned}
$$

and finally divide, to get:

$P(asylum \mid n \land \neg k) = \frac{P(asylum \land n \land \neg k)}{P(n \land \neg k)} = \frac{.0032791}{.26541} = .012354$ (or 1.24%)

(b) Okay, now let's achieve the same result through the normalization method. Now, we have to find all three posterior location numerators, then scale them to sum to 1. To wit:

$$
\begin{aligned}
P(foster \wedge n \wedge \neg k) = {} & P(foster \wedge n \wedge \neg k \wedge e \wedge r) + \\
& P(foster \wedge n \wedge \neg k \wedge e \wedge \neg r) + \\
& P(foster \wedge n \wedge \neg k \wedge \neg e \wedge r) + \\
& P(foster \wedge n \wedge \neg k \wedge \neg e \wedge \neg r) \\
= {} & P(e) \cdot P(r \mid e) \cdot P(foster \mid r \wedge e) \cdot P(n \mid foster) \cdot P(\neg k \mid foster \wedge n) + \\
& P(e) \cdot P(\neg r \mid e) \cdot P(foster \mid \neg r \wedge e) \cdot P(n \mid foster) \cdot P(\neg k \mid foster \wedge n) + \\
& P(\neg e) \cdot P(r \mid \neg e) \cdot P(foster \mid r \wedge \neg e) \cdot P(n \mid foster) \cdot P(\neg k \mid foster \wedge n) + \\
& P(\neg e) \cdot P(\neg r \mid \neg e) \cdot P(foster \mid \neg r \wedge \neg e) \cdot P(n \mid foster) \cdot \\
& P(\neg k \mid foster \wedge n) \\
= {} & .1 \cdot .8 \cdot .2 \cdot .9 \cdot .99 + \\
& .1 \cdot .2 \cdot .2 \cdot .9 \cdot .99 + \\
& .9 \cdot .001 \cdot .2 \cdot .9 \cdot .99 + \\
& .9 \cdot .999 \cdot .3 \cdot .9 \cdot .99 \\
= {} & .25831
\end{aligned}
$$

and:

$$
\begin{aligned}
P(atlarge \wedge n \wedge \neg k) = {} & P(atlarge \wedge n \wedge \neg k \wedge e \wedge r) + \\
& P(atlarge \wedge n \wedge \neg k \wedge e \wedge \neg r) + \\
& P(atlarge \wedge n \wedge \neg k \wedge \neg e \wedge r) + \\
& P(atlarge \wedge n \wedge \neg k \wedge \neg e \wedge \neg r) \\
= {} & P(e) \cdot P(r \mid e) \cdot P(atlarge \mid r \wedge e) \cdot P(n \mid atlarge) \cdot P(\neg k \mid atlarge \wedge n) + \\
& P(e) \cdot P(\neg r \mid e) \cdot P(atlarge \mid \neg r \wedge e) \cdot P(n \mid atlarge) \\
& \cdot P(\neg k \mid atlarge \wedge n) + \\
& P(\neg e) \cdot P(r \mid \neg e) \cdot P(atlarge \mid r \wedge \neg e) \cdot P(n \mid atlarge) \\
& \cdot P(\neg k \mid atlarge \wedge n) + \\
& P(\neg e) \cdot P(\neg r \mid \neg e) \cdot P(atlarge \mid \neg r \wedge \neg e) \cdot P(n \mid atlarge) \\
& \cdot P(\neg k \mid atlarge \wedge n) \\
= {} & .1 \cdot .8 \cdot .2 \cdot .2 \cdot .05 + \\
& .1 \cdot .2 \cdot .3 \cdot .2 \cdot .05 + \\
& .9 \cdot .001 \cdot .6 \cdot .2 \cdot .05 + \\
& .9 \cdot .999 \cdot .4 \cdot .2 \cdot .05 \\
= {} & .003822
\end{aligned}
$$

We now have the vector $< .003279, .25831, .003822 >$, which, when normalized (by dividing each entry by the sum of the entries, or .26541) gives us:

$$\mathbb{P}(T) = < asylum = .012354, foster = .97325, atlarge = .01440 > \ \checkmark$$

Either way: clearly that bit about the niceness and the dog makes us *lots* less enthusiastic about the asylum hypothesis.