Name: ________________________________

DATA 470 — Fall 2025

Quiz #2 (Sep. 2, 2025)

1. Give three different linear combinations of the following vectors: $\begin{bmatrix} 3 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 1 \end{bmatrix}$.

   Sure, how about these:

   - $\begin{bmatrix} 3 & 2 \end{bmatrix}$ (one copy of the first plus zero of the second)
   - $\begin{bmatrix} -18 & -18 \end{bmatrix}$ (zero copies of the first plus -18 of the second)
   - $\begin{bmatrix} 7 & 5 \end{bmatrix}$ (two copies of the first plus one of the second)

2. What is $\begin{bmatrix} 4 & 0 & 1 & 1 & 0 & 19 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 0 \end{bmatrix}$, or is that operation not even possible?

   It's possible. The answer is **11**. (Important: the answer is a scalar, not a vector!)

3. Are these two vectors orthogonal? Why or why not? $\begin{bmatrix} 5 & 2 \end{bmatrix}$ and $\begin{bmatrix} -2 & -5 \end{bmatrix}$

   No. Their dot product is $5 \cdot -2 + 2 \cdot -5 = -20 \neq 0$. So they are not at right angles to each other.

4. Are these two vectors orthogonal? Why or why not? $\begin{bmatrix} 5 & -2 \end{bmatrix}$ and $\begin{bmatrix} -2 & -5 \end{bmatrix}$

   Yes. Their dot product is $5 \cdot -2 + -2 \cdot -5 = 0$. So they *are* at right angles to each other.

5. Normalize this vector (using the Euclidean (ℓ^2) norm): $\begin{bmatrix} 2 & 1 & 2 \end{bmatrix}$

   First, calculate the norm: $\sqrt{2^2 + 1^2 + 2^2} = \sqrt{9} = 3$. Then, divide each entry to the norm to get: $\begin{bmatrix} \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \end{bmatrix}$.

6. Is the vector you gave in your answer to question 5 normalized? (Sure hope so!) Verify that it is.

   The norm is $\sqrt{\frac{2}{3}^2 + \frac{1}{3}^2 + \frac{2}{3}^2} = \sqrt{\frac{4}{9} + \frac{1}{9} + \frac{4}{9}} = \sqrt{1} = 1. \checkmark$

7. Are these vectors linearly independent? Why or why not? $\begin{bmatrix} 3 & -2 \end{bmatrix}$ and $\begin{bmatrix} 6 & -4 \end{bmatrix}$

   No way. One is a multiple of the other! (Two of the first vector in the set gives the second set. So they point in exactly the same direction.)